Physics - Formulary

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Contents

1 Kinematics

1.1 Translational Motion

s	m	distance, way travelled
v	m/s	velocity
a	m/s^2	acceleration
t	s	time
s_0	m	initial position
v_0	m/s	initial velocity
a_0	m/s^2	initial acceleration
e		unit vector

Uniform Movement

Conditions: $a = 0 \quad ; \quad v \neq 0 = \text{const.}$

$$v = \int a \; \mathrm{d}t = v_0 = \text{const.}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t}$$

$$s = \int v \; \mathrm{d}t = v_0 \, t + s_0$$

Uniformly accelerated Motion

Conditions: $a = a_0 > 0 = \text{const.}$

$$v = \int a_0 \; \mathrm{d}t = a_0 \, t + v_0$$

$$s = \int v \; \mathrm{d}t = \int (a_0 \, t + v_0) \; \mathrm{d}t = \frac{a_0}{2} \, t^2 + v_0 \, t + s_0$$

Irregular accelerated Motion

Conditions: $a = a(t) \neq \text{const.}$

$$v = \int a(t) \; \mathrm{d}t = a(t) \, t + v_0$$

$$s = \int v(t) \; \mathrm{d}t = \int (a(t) \, t + v_0) \; \mathrm{d}t = \frac{a(t)}{2} \, t^2 + v_0 \, t + s_0$$

Translational Motion in general

$$s = \int_{t_1}^{t_2} v(t)\ \mathrm{d}t$$

$$v = \int_{t_1}^{t_2} a(t)\ \mathrm{d}t \qquad v = \frac{\mathrm{d}s}{\mathrm{d}t} = s'(t) = \dot{s}$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = v'(t) = \dot{v} \qquad a = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = s''(t) = \ddot{s}$$

Acceleration with $v_0 = 0$

$$s = \frac{a}{2}\,t^2 \qquad s = \frac{v}{2}\,t$$

$$v = a\,t \qquad v = \sqrt{2\,a\,s}$$

Acceleration with $v_0 \neq 0$

$$s = \frac{a}{2}\,t^2 + v_0\,t \qquad s = \frac{v + v_0}{2}\,t$$

$$v = a\,t + v_0 \qquad v = \sqrt{2\,a\,s + v_0^2}$$

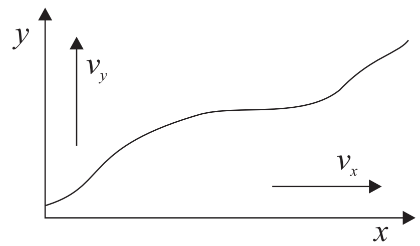
Average velocity

$$\overline{v} = \frac{\Delta s}{\Delta t}$$

Average acceleration

$$\overline{a} = \frac{\Delta v}{\Delta t}$$

2D-Translational Motion



To calculate separately

$$s(t) = \begin{matrix} x(t) \\ y(t) \end{matrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x(t) = v_x(t)\,t + x_0 \qquad y(t) = v_y(t)\,t + y_0$$

3D-Translational Motion

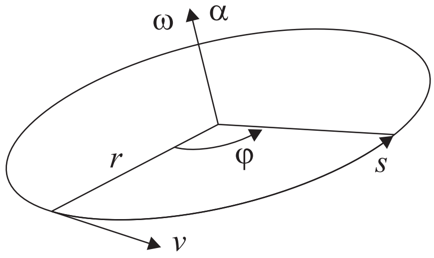
$$\vec{s}(t) = x(t)\,\vec{e}_x + y(t)\,\vec{e}_y + z(t)\,\vec{e}_z$$

$$\vec{v}(t) = v_x(t)\,\vec{e}_x + v_y(t)\,\vec{e}_y + v_z(t)\,\vec{e}_z$$

$$\vec{a}(t) = a_x(t)\,\vec{e}_x + a_y(t)\,\vec{e}_y + a_z(t)\,\vec{e}_z$$

1.2 Rotational Motion

r	r	radius
φ	rad	angle in radians
ω	s^{-1}	angular velocity
α	s^{-2}	angular acceleration
t	s	time
T	s	period, duration of a rotation
f	s^{-1}, Hz	revolution frequency
n	s^{-1}	rotational speed



Uniform Rotation

Condition: $\omega = \text{const.}$

$$\varphi = \omega\, t + \varphi_0$$

$$\omega = \frac{2\,\pi}{T} = 2\,\pi\, f \qquad n = \frac{1}{T}$$

Uniformly accelerated Rotation

Condition: $\alpha = \text{const.}$

$$\varphi = \frac{\alpha}{2}\, t^2 + \omega_0\, t + \varphi_0$$

Rotation in general

$$\varphi = \int_{t_1}^{t_2} \omega(t)\ \mathrm{d}t$$

$$\omega = \int_{t_1}^{t_2} \alpha(t)\ \mathrm{d}t \qquad \omega = \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \varphi'(t) = \dot{\varphi}$$

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \omega'(t) \qquad \alpha = \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = \varphi''(t) = \ddot{\varphi}$$

1.3 Relationship Translational - Rotational Motion

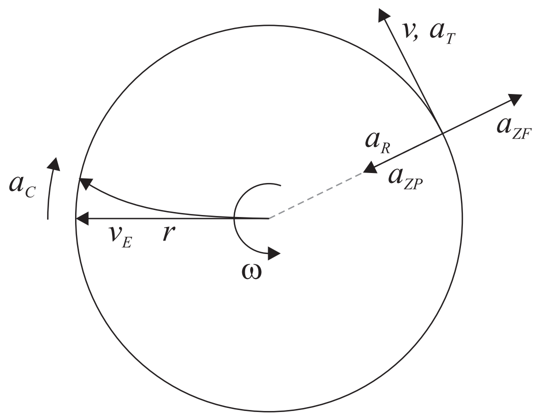
r	r	radius
φ	rad	angle in radians
ω	s^{-1}	angular velocity
α	s^{-2}	angular acceleration
t	s	time
s	m	length of a circular arc
v	m/s	path velocity
v_E	m/s	own speed at the rotation body
a	m/s^2	acceleration
a_{ZP}	m/s^2	centripetal acceleration
a_{ZF}	m/s^2	centrifugal acceleration
a_R	m/s^2	radial acceleration
a_T	m/s^2	tangential acceleration
a_C	m/s^2	coriolis acceleration
e		unit vector

Overview

$$s = \varphi\, r \qquad \vec{s} = \varphi\, \vec{e}_\omega \times \vec{r}$$

$$v = \omega\, r \qquad \vec{v} = \vec{\omega} \times \vec{r}$$

$$s = \alpha\, r \qquad \vec{a} = \vec{\alpha} \times \vec{r}$$



Centripetal Acceleration, Radial Acceleration

Acceleration is directed towards center

$$a_{ZP} = a_R = -\omega^2\, r$$

$$\vec{a}_{ZP} = \vec{\omega}^2 \times \vec{r} \qquad \vec{a}_{ZP} = \frac{-v^2}{r}$$

Uniformly accelerated rotation

$$a_{ZP} = r\,\left[\,\omega_0 + \alpha_0\,\left(t - t_0\right)\right]^2$$

Centrifugal Acceleration

$$a_{ZF} = -a_{ZP}$$

Tangential Acceleration

Uniformly accelerated rotation

$$a_T = \alpha_0\,r$$

Uniform rotation

$$a_T = 0$$

Coriolis Acceleration

$$a_C = -2\,\left(\vec{\omega} \times \vec{v}_E\,\right) \qquad a_C = -2\,\omega\,v_E$$

2 Dynamics

2.1 Newton's Law

F	N	force
v	m/s	velocity
m	kg	mass
a	m/s^2	acceleration
t	s	time

Inertia Principle

Condition: as long as there acts no force $F = 0$

$$\sum_{i=1}^{n} F_i = 0$$

$$v = 0 \qquad v = \text{const.}$$

Action Principle

A force F applied at a mass m generates an acceleration a.
Condition: $m = \text{const.}$

$$F = m\,a$$

Conditions: $m = m(t) \neq \text{const.}$

$$F = \frac{\mathrm{d}}{\mathrm{d}t}(m\,v) \qquad F = \frac{\mathrm{d}}{\mathrm{d}t}(m)\,v + \frac{\mathrm{d}}{\mathrm{d}t}(m\,v)$$

$$F = \dot{\mathbf{m}}\,v + m\,a$$

Reaction Principle

actio = reactio

$$|\vec{F}_1| = |-\vec{F}_2|$$

2.2 Dynamics of Translational Motion

F	N	force
v	m/s	velocity
m	kg	mass
a	m/s^2	acceleration
t	s	time
p	Ns	momentum
I	Ns	force impact

Force

$$F = m\,a \qquad a = \frac{F}{m} = \frac{\mathrm{d}v}{\mathrm{d}t} \qquad \mathrm{d}v = \frac{F}{m}\,\mathrm{d}t$$

$$\int\limits_{t_1}^{t_2} \vec{F}\ \mathrm{d}t = \int\limits_{t_1}^{t_2} m\,\vec{a}\ \mathrm{d}t \qquad \int\limits_{t_1}^{t_2} \vec{F}\ \mathrm{d}t = \int\limits_{t_1}^{t_2} m\,\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\ \mathrm{d}t$$

$$\int\limits_{t_1}^{t_2} \vec{F}\ \mathrm{d}t = m \int\limits_{v_1}^{v_2} \mathrm{d}\vec{v} = m\ (\vec{v}_2 - \vec{v}_1)$$

Momentum

$$p = m\,v$$

$$\vec{p} = \int \mathrm{d}\vec{p} = m\,\vec{v} \qquad \mathrm{d}\vec{p} = m\ \mathrm{d}\vec{v}$$

Force Impact

A Force Impact exerts a Momentum.

$$I = F\ \Delta t = m\ \Delta v = \Delta p$$

Force is the temporal change of Momentum.

$$F = m\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}p}{\mathrm{d}t}$$

Conservation of Momentum

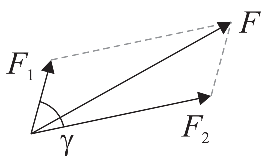
$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots = \vec{p}_{ges} = \sum_{i=1}^{n} \vec{p}_i$$

$$m_1\,\vec{v}_1 + \int\limits_{1_1}^{t_2} \vec{F}\ \mathrm{d}t = m_2\,\vec{v}_2$$

2.3 Force

F	N	force
F_G	N	weight force
F_H	N	downward force
F_N	N	normal force
F_R	N	friction force
μ_{GR}		coefficient of sliding friction
μ_{HR}		coefficient of static friction
μ_{RR}		coefficient of rolling friction
γ	rad	angle
v	m/s	velocity
m	kg	mass
a	m/s^2	acceleration
g	m/s^2	gravitational acceleration
ω	s^{-1}	angular velocity
r	m	radius
M	Nm	torque

Compound Forces



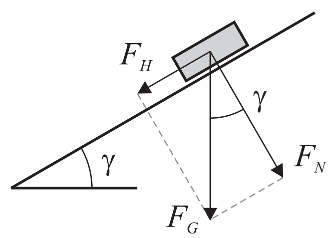
$$\gamma = \angle\left(\vec{F}_1, \vec{F}_2\right)$$

$$F = \sqrt{F_1^2 + F_2^2 + 2\,F_1\,F_2\,\cos\gamma}$$

Condition: $F_1 = F_2$

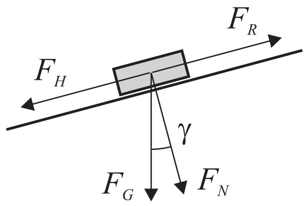
$$F_1 = F_2 = \frac{F}{\sqrt{2\,(1+\cos\gamma)}}$$

Inclined Plane



$$\sin\gamma = \frac{F_H}{F_G} \qquad \cos\gamma = \frac{F_N}{F_G} \qquad \tan\gamma = \frac{F_H}{F_N}$$

Solid State Friction



Sliding friction (motion)
Condition: $F_H > F_R$

$$F_R = \mu_{GR}\, F_N \qquad F_R = \mu_{GR}\, F_G\, \cos\gamma$$

Static friction (motionless)
Condition: $F_H \leq F_R$

$$F_R = \mu_{HR}\, F_N$$

Rolling friction

$$F_R = \mu_{RR}\, \frac{F_N}{2\,r}$$

Types of Forces

Weight force

$$F_G = m\, g$$

Normal force

$$F_N = F_G\, \cos\gamma$$

Friction force

$$F_R = \mu_{GR}\, F_N$$

Downward force

$$F_N = F_G\, \sin\gamma$$

Centripetal force

$$\vec{F}_{ZP} = -m\, \omega^2\, \vec{r}$$

Coriolis force

$$\vec{F}_C = -2\, m\, \vec{\omega} \times \vec{v}$$

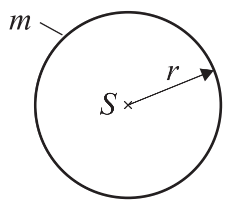
Tangential force

$$\vec{F}_T = \vec{M} \times \vec{r}$$

2.4 Dynamics of Rotational Motion

M	Nm	torque
J	$\mathrm{kg\,m^2}$	moment of inertia
J_S	$\mathrm{kg\,m^2}$	moment of inertia with axis of rotion at center S
J_A	$\mathrm{kg\,m^2}$	moment of inertia with axis of rotion at A
m	kg	mass
ρ	$\mathrm{kg/m^3}$	density
V	$\mathrm{m^3}$	volume
r	m	radius
F	N	force
L	$\mathrm{kg\,m^2/s}$	angular momentum
p	$\mathrm{kg\,m/s}$	momentum
I_D	$\mathrm{kg\,m^2/s}$	torque impact
ω	$\mathrm{s^-1}$	angular velocity
α	$\mathrm{s^-2}$	angular acceleration
v	m/s	velocity
s	m	distance
t	s	time

Rotation around Priority Axis



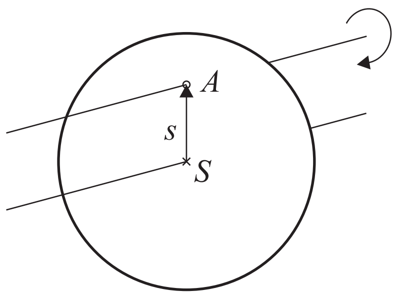
$$J = \rho \int_0^{V_{ges}} r^2 \,\mathrm{d}V$$

$$J = \rho \int_0^{m_{ges}} r^2 \,\mathrm{d}m \qquad J = \sum_{i=0}^{n} r_i^2 \,\Delta m \qquad J = r^2 \,\Delta m$$

Special case:

$$J_S = r^2 \, m$$

Rotation around Axis parallel to Priority Axis

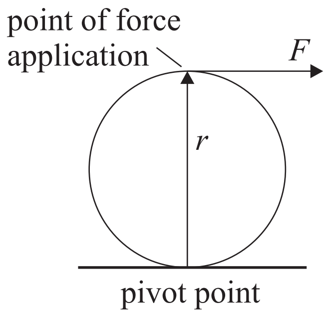


Steiner's sentence:

$$J_A = J_S + m\,s^2$$

$$J = \rho \int\int\int r^2(x, y, z)\ \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z$$

Torque



$$M = J\,\alpha \qquad M = \alpha \int r^2\ \mathrm{d}m \qquad M = \vec{r} \times \vec{F}$$

$$M = \left|\vec{F}\right|\ |\vec{r}| \sin\angle\left(\vec{r}, \vec{F}\right) = \left|\vec{F_{\tan}}\right|\ |\vec{r}|$$

Angular Momentum

$$L = J\,\omega$$

$$L = \vec{r} \times \vec{p} \qquad L = m\ (\vec{r} \times \vec{v})$$

$$L = m\ (\vec{r} \times (\vec{\omega} \times \vec{r})) \qquad L = m\,\vec{r}^2\,\vec{\omega} - (\vec{r}\,\vec{\omega})\ \vec{r}$$

$$M = \frac{\mathrm{d}L}{\mathrm{d}t} \qquad M = \frac{\mathrm{d}\,(\vec{r} \times \vec{p})}{\mathrm{d}t} \qquad M = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \times m\,\vec{v} + \vec{r} \times \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

Torque Impact

A Torque Impact exerts an angular Momentum.

$$I_D = \Delta L$$

$$I_D = \int M \, \mathrm{d}t \qquad I_D = \int_{t_1}^{t_2} M \, \mathrm{d}t = I_D = \int_{t_1}^{t_2} J \, \alpha \, \mathrm{d}t$$

$$I_D = \int_{t_1}^{t_2} J \, \frac{\mathrm{d}\omega}{\mathrm{d}t} \qquad I_D = L \, (\omega_2 - \omega_1) = J \, \Delta\omega$$

Conservation of Angular Momentum

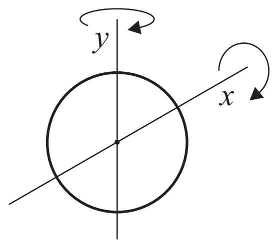
$$\vec{L}_1 + \int_{t_1}^{t_2} \vec{M} \, \mathrm{d}t = \vec{L}_2$$

$$L = L_1 + L_2 + \ldots + L_n = \text{const.}$$

2.5 Selected Moment of Inertia

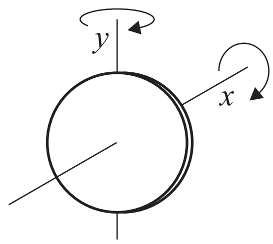
J	$\mathrm{kg\,m^2}$	moment of inertia
J_S	$\mathrm{kg\,m^2}$	moment of inertia with axis of rotion at center S
m	kg	mass
r	m	radius
h	m	height
l	m	length

Circular Ring, thin



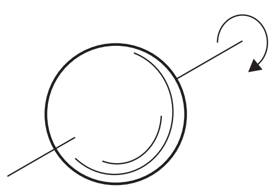
$$x: \quad J_S = m\,r^2 \qquad y: \quad J_S = \frac{m}{2}\,r^2$$

Circular Disc, thin



$$x: \quad J_S = \frac{m}{2}\,r^2 \qquad y: \quad J_S = \frac{m}{4}\,r^2$$

Ball



Solid ball

$$J_S = \frac{2}{5}\,m\,r^2$$

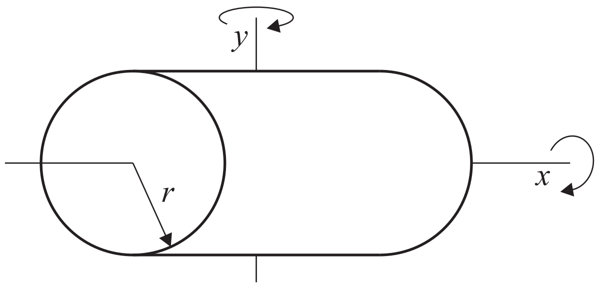
Hollow ball

$$J_S = \frac{2}{5}\,m\,\frac{r_a^5 - r_r^5}{r_a^3 - r_r^3}$$

Hollow ball, thin-walled

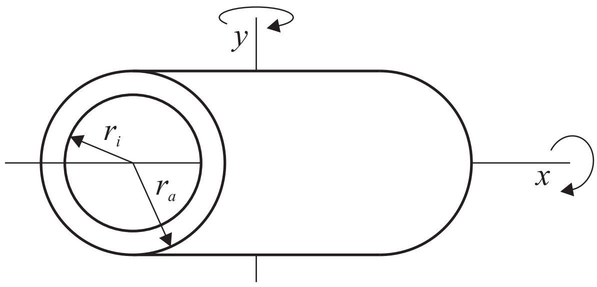
$$J_S = \frac{2}{3}\, m\, r^2$$

Cylinder



$$x: \quad J_S = \frac{m}{2}\, r^2 \qquad y: \quad J_S = \frac{m}{12}\left(3\, r^2 + h^2\right)$$

Hollow Cylinder

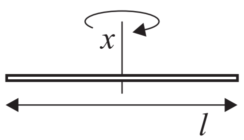


$$x: \quad J_S = \frac{m}{2}\left(r_i^2 + r_a^2\right) \qquad y: \quad J_S = \frac{m}{4}\left(r_i^2 + r_a^2 + \frac{h^2}{3}\right)$$

Thin-walled: $r_i \approx r_a$

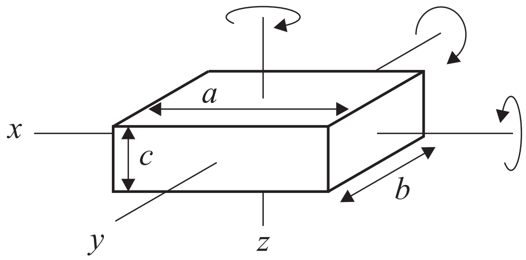
$$x: \quad J_S = m\, r^2 \qquad y: \quad J_S = \frac{m}{4}\left(2\, r^2 + \frac{h^2}{3}\right)$$

Rod, long and thin



$$x: \quad J_S = \frac{m}{12}\, l^2$$

Cuboid

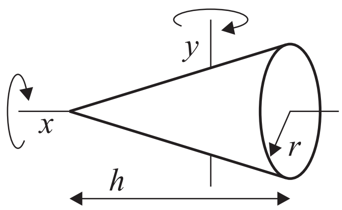


$$x: \quad J_S = \frac{m}{12}\left(b^2 + c^2\right)$$

$$y: \quad J_S = \frac{m}{12}\left(a^2 + c^2\right)$$

$$z: \quad J_S = \frac{m}{12}\left(a^2 + b^2\right)$$

Cone



$$x: \quad J_S = \frac{3}{10}\, m\, r^2$$

$$y: \quad J_S = \frac{3}{20}\, m \left(r^2 + \frac{h^2}{4}\right)$$

2.6 Calculation of Center of Gravity

r_{SP}	m	vector of the center of gravity
d	m	direction vector
m	kg	mass
V	m^3	volume
ρ	$\mathrm{kg/m}^3$	density

Center of Gravity of a Continuous Object

$$\vec{r}_{SP} = \frac{\int\limits_0^m \vec{d}\,\mathrm{d}m}{\int\limits_0^m \mathrm{d}m} = \begin{pmatrix} x_{SP} \\ y_{SP} \\ z_{SP} \end{pmatrix}$$

$$x_{SP} = \frac{\int\limits_0^m x\,\mathrm{d}m}{\int\limits_0^m \mathrm{d}m} \qquad \begin{matrix} y_{SP} = \dots \\ z_{SP} = \dots \end{matrix}$$

Center of Gravity of n Pointmasses

$$\vec{r}_{SP} = \frac{\sum\limits_{i=0}^{n} m_i\, d_i}{\sum\limits_{i=0}^{n} m_i} = \begin{pmatrix} x_{SP} \\ y_{SP} \\ z_{SP} \end{pmatrix}$$

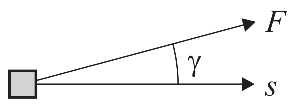
$$x_{SP} = \frac{\sum\limits_{i=0}^{n} m_i\, x_i}{\sum\limits_{i=0}^{n} m_i} \qquad \begin{matrix} y_{SP} = \dots \\ z_{SP} = \dots \end{matrix}$$

3 Work and Energy

3.1 Work

W	J, $\mathrm{kg\,m^2/s^2}$	work
F	N	force
F_G	N	weight force
F_B	N	accelerating force
F_N	N	normal force
F_R	N	friction force
F_A	N	external force
s	m	distance, way travelled
h	m	height
γ	rad	angle
a	$\mathrm{m/s^2}$	acceleration
g	$\mathrm{m/s^2}$	gravitational acceleration
v	m/s	velocity
k	N/m	spring constant
φ	rad	angle of rotation
ω	$\mathrm{s^{-1}}$	angular velocity
α	$\mathrm{s^{-2}}$	angular acceleration
J	$\mathrm{kg\,m^2}$	moment of inertia
r	m	distance to center of gravity
f	$\mathrm{Nm^2/kg^2}$	gravitational constant

General



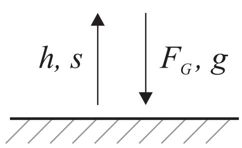
$$W = F\,s\,\cos\gamma$$

Special case:

$$W = F\,s$$

$$W = \int_{s_1}^{s_2} F\,\cos\gamma\,\mathrm{d}s \qquad W = \int_{s_1}^{s_2} \vec{F}(s)\,\mathrm{d}s$$

Lifting Work



$$W_{HUB} = -\int F_G \, \mathrm{d}s$$

$$W_{HUB} = F_G \, \Delta s \, \cos\gamma \qquad W_{HUB} = -m \, g \, \Delta s \, \cos\gamma$$

Condition: $\gamma = \pi = 180\text{deg}$

$$W_{HUB} = m \, g \, \Delta h$$

Accelerating Work

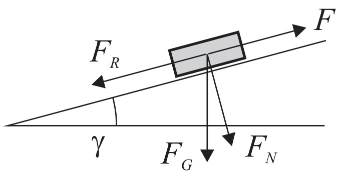
$$W_B = m \, a \, s \qquad W_B = \frac{m}{2} \, v^2 \qquad W_B = \frac{m}{2} \left(v^2 - v_0^2\right)$$

$$W_B = -\int \vec{F}_B \, \mathrm{d}s \qquad W_B = \vec{F}_B \, s$$

$$W_B = m \int a \, \mathrm{d}s \qquad W_B = m \int \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} \, \mathrm{d}s$$

$$W_B = m \int \frac{\mathrm{d}\vec{s}}{\mathrm{d}t} \, \mathrm{d}v \qquad W_B = m \int \vec{v} \, \mathrm{d}\vec{v}$$

Frictional Work



$$W_R = F_R \, s \qquad W_R = \mu \, F_N \, s \qquad W_R = \mu \, F_G \, s \, \cos\gamma$$

$$W_R = \int \vec{F}_R \, \mathrm{d}\vec{s}$$

Condition: $\gamma = 0$

$$W_R = \mu \, m \, g \, s$$

Spring Tension Work

$$W_F = \frac{1}{2} \, k \, s^2 \qquad W_F = \int \vec{F} \, \mathrm{d}s \qquad W_F = \int_{s_{min}}^{s_{max}} k \, s \, \mathrm{d}s$$

Rotation, torsion:

$$W_F = \frac{1}{2} \, k \, \varphi^2 \qquad W_F = \frac{1}{2} \, k \, \left(\varphi_2^2 - \varphi_1^2\right)$$

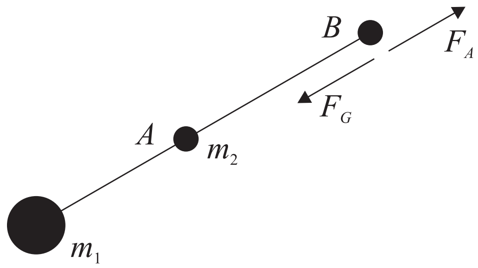
Rotational Work

$$W_{ROT} = \int\limits_{\varphi_0}^{\varphi_1} M(\varphi)\, \mathrm{d}\varphi$$

$$W_{ROT} = \int J\, \alpha\, \mathrm{d}s \qquad W_{ROT} = \int J\, \frac{\mathrm{d}\omega}{\mathrm{d}t}\, \mathrm{d}s \qquad W_{ROT} = J \int \frac{\mathrm{d}s}{\mathrm{d}t}\, \mathrm{d}\omega$$

$$W_{ROT} = J \int \omega\, \mathrm{d}\omega \qquad W_{ROT} = \frac{1}{2}\, J\, \omega^2 \qquad W_{ROT} = \frac{1}{2}\, J\, \left(\omega_1^2 - \omega_0^2\right)$$

Gravitational Work



Gravitational work is executed when the smaller mass m_2 is lifted from A to B (lifting work).

$$W_{AB} = \int\limits_{r_1}^{r_2} F_A\, \mathrm{d}r \qquad W_{AB} = -\int\limits_{r_1}^{r_2} F_G\, \mathrm{d}r$$

$$W_{AB} = \int\limits_{r_1}^{r_2} f\, m_1\, m_2\, \frac{1}{r^2}\, \mathrm{d}r \qquad W_{AB} = f\, m_1\, m_2\, \left(\frac{1}{r} - \frac{1}{r}\right)$$

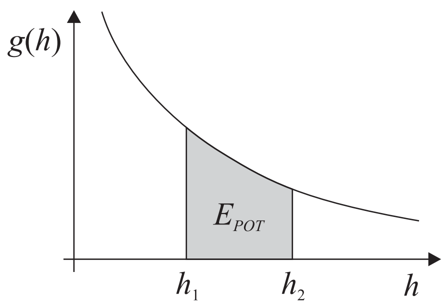
Gravitational constant:

$$f = 6.673 \cdot 10^{-11}\, \frac{N\, m^2}{kg^2}$$

3.2 Energy

E	J	energy
E_V	J	energy loss
m	kg	mass
s	m	distance, way travelled
h	m	height
v	m/s	velocity
k	N/m	spring constant
J	kg m^2	moment of inertia
ω	s^{-1}	angular velocity

Potential Energy



$$E_{POT} = m \int_{h_1}^{h_2} g \, \mathrm{d}h$$

Condition: $g = \text{const.}$

$$E_{POT} = m \, g \, h$$

Kinetic Energy

$$E_{KIN} = \frac{m}{2} v^2 \qquad E_{KIN} = \frac{m}{2} \left(v_1^2 - v_0^2\right)$$

Tensional Energy

Potential energy:

$$E_{POT} = \frac{1}{2} k \, s^2$$

Rotational Energy

$$E_{ROT} = \frac{1}{2} J \, \omega^2$$

Rolling wheel:

$$E_{KIN} = E_{ROT} + E_{TRANS}$$

Principle of Conservation of Energy

$$E = \sum_{i=1}^{n} E_i = \text{const.}$$

Condition: conservative forces (lossless):

$$\sum E_{POT} + \sum E_{KIN} = \text{const.}$$

Condition: dissipative forces (loss due to friction):

$$\sum E_{POT}|_{t_0} + \sum E_{KIN}|_{t_0} = \sum E_{POT}|_{t_1} + \sum E_{KIN}|_{t_1} + E_V$$

Mechanical Impacts

Elastic impact:

$$E_{KIN1}(t_1) + E_{KIN2}(t_1) = E_{KIN1}(t_2) + E_{KIN2}(t_2)$$

$$\frac{m_1}{2}\,v_1^2 + \frac{m_2}{2}\,v_2^2 = \frac{m_1}{2}\,v_1'^2 + \frac{m_2}{2}\,v_2'^2$$

Inelastic collision:

$$E_{KIN1}(t_1) + E_{KIN2}(t_1) = E_{KIN}(t_2) + E_V$$

$$\frac{m_1}{2}\,v_1^2 + \frac{m_2}{2}\,v_2^2 = \frac{m_1 + m_2}{2}\,v'^2 + E_V$$

$$E_V = E_1 - E_2 \qquad E_V = \frac{m_1\,m_2}{2\,(m_1 + m_2)}\,(v_1 - v_2)^2$$

3.3 Power

P	W	power
η		efficiency
W	J	work
E	J	energy
F	N	force
v	m/s	velocity
s	m	distance, way travelled
t	s	time

Power

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} \qquad P = \vec{F}\,\frac{\mathrm{d}\vec{s}}{\mathrm{d}t} \qquad P = \vec{F}\,\vec{v}$$

Average power

$$P_m = \frac{W_{tot}}{t_{tot}}$$

Efficiency

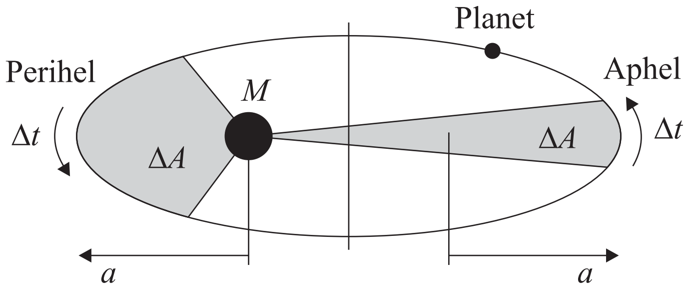
$$\eta = \frac{W_{out}}{W_{in}} \qquad \eta = \frac{E_{out}}{E_{in}} \qquad \eta = \frac{P_{out}}{P_{in}}$$

$$\eta = \frac{\int_0^{t_{in}} P_{out}\,\mathrm{d}t}{\int_0^{t_{in}} P_{in}\,\mathrm{d}t} \qquad \eta = \prod_{i=1}^{n} \eta_i = \eta_1\ \eta_2\ \eta_3\ \ldots$$

3.4 Gravity

t	s	time
T	s	period of circulation
a	m	half-axis of a planet
A	m^2	swept area
F_G	N	weight force
m	kg	mass
m_g	kg	mass of the earth
f	$m^3/kg\,s^2$	gravitational constant
r	m	radius, distance between two pointmasses
e		unit vector
g	m/s^2	gravitational acceleration
ϕ	m^2/s^2	gravitational potential
W	J	work
E	J	energy

Kepler's Laws of Planetary Motion



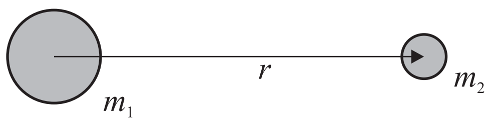
Second law of Kepler

$$\frac{\Delta A}{\Delta t} = \text{const.}$$

Third law of Kepler

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

Newton's Law of Gravity



$$\vec{F}_G = -f\,\frac{m_1\,m_2}{r^2}\,\vec{e}_r \qquad f = 6.673\cdot 10^{-11}\,\frac{N\,m^2}{kg^2}$$

Condition: $r \leq r_E$

$$g = -f\,\frac{m_E}{r^2}\,\vec{e}_r$$

Lifting Work and Potential Energy

$$W = -f\,m_E\,m_K\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad W_M = -f\,\frac{m_E}{r}\,m$$

$$W = \phi\,m \qquad \phi = -f\,\frac{m_E}{r} \qquad W = m\,(\phi_2 - \phi_2)$$

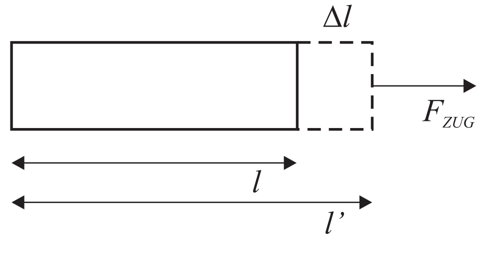
$$\phi = -\int_{\infty}^{r} \vec{g}(r)\,\mathrm{d}\vec{r} \qquad \nabla\phi = -\vec{E}$$

4 Mechanics and Oscillation

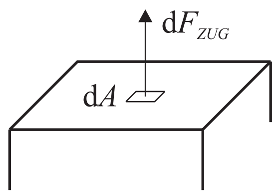
4.1 Deformable Body Mechanics

σ	N/m^2	tension
ε		elongation
ε_q		lateral strain
E	N/m^2	modulus of elasticity (material value)
F_{ZUG}	N	traction force, normal force
F_T	N	shearing force (tangential)
A	m^2	area
V	m^3	volume
l	m	initial length, length of a body
l'	m	final length
d	m	initial thickness
d'	m	final thickness
μ		Poisson's ratio
ν		inverse Poisson's ratio, transverse direction
K	N/m^2	compressive modulus
κ	Pa^{-1}	compressibility
Δp	N/m^2	change of pressure
G	N/m^2	shear modulus, modulus of torsion
τ	N/m^2	shear stress
γ	rad	shift, shear, shear angle
φ	rad	torsion angle
r	m	radius
M	Nm	torque
M_T	Nm	torque in torsion
D	Nm/rad	torsional coefficient, spring constant
I_{POL}	rad/m^4	polar moment of inertia

Elongation



$$\Delta l = |l - l'| \qquad \varepsilon = \frac{\Delta l}{l}$$



$$\sigma = \frac{F_{ZUG}}{A} \qquad \sigma = \frac{\mathrm{d}F_{ZUG}}{\mathrm{d}A}$$

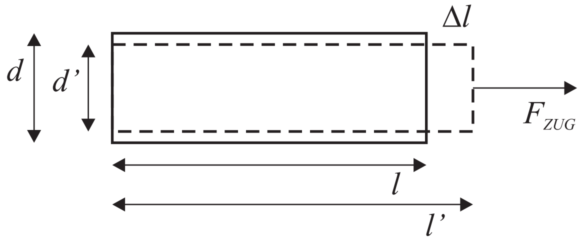
Hooke's law, for the elastic range only:

$$\sigma = E\,\epsilon$$

$$E(\sigma) = \frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon} \qquad E = \sigma\,\frac{l}{\Delta l} \qquad E = \frac{F_{ZUG}}{A}\,\frac{l}{\Delta l}$$

Lateral Strain

Change of length and thickness.



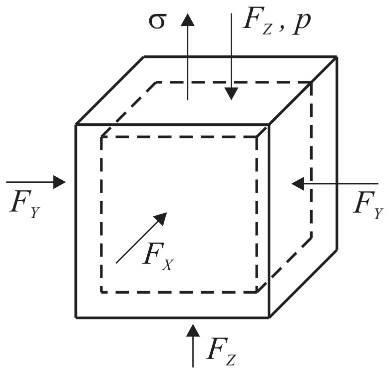
$$\Delta d = d - d' \qquad \mu = -\frac{\frac{\Delta d}{d}}{\frac{\Delta l}{l}} \qquad \mu = \frac{1}{\nu}$$

$$\varepsilon_q = -\frac{1}{\mu}\,\varepsilon \qquad \varepsilon_q = -\nu\,\varepsilon \qquad \varepsilon_q = \frac{\Delta d}{d}$$

Relative change of volume:

$$\frac{\Delta V}{V} = \varepsilon\,(1 - 2\,\nu) \qquad \frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{2\,\Delta d}{d}$$

All sides Compression



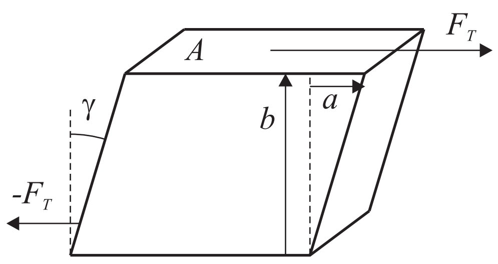
$$\frac{\Delta V}{V} = \frac{\sigma}{K} \qquad \frac{\Delta V}{V} = \frac{\Delta p}{K}$$

$$\mathrm{d}V = -\frac{1}{K}\, V\, \mathrm{d}p$$

$$K = \frac{1}{\kappa} \qquad K = \frac{E}{3\,(1 - 2\,\nu)} \qquad \frac{\Delta V}{V} = 3\,\varepsilon\,(1 - 2\,\nu)$$

$$\nu = \frac{1}{\mu} \qquad \varepsilon = \frac{\sigma}{E} \qquad \sigma = -\Delta p$$

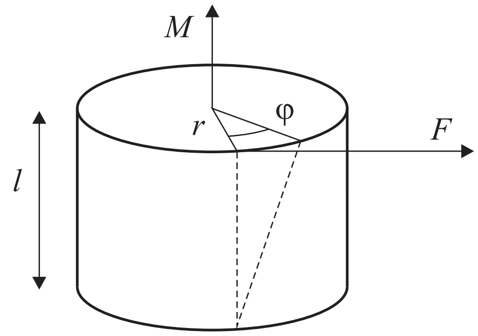
Shear



$$\tau = G\,\gamma \qquad \tau = \frac{F_T}{A} \qquad \tau = \frac{G\,a}{d}$$

$$G(\tau) = \frac{\mathrm{d}\tau}{\mathrm{d}\gamma} \qquad G = \frac{E}{2\,(1+\nu)}$$

Torsion



$$\varphi = \frac{2\,l\,M}{\pi\,G\,r^4} \qquad D = \frac{\pi\,G\,r^4}{2\,l}$$

$$M = D\,\varphi \qquad \vec{M} = D\,\varphi\,\vec{e}_\omega$$

$$\ddot{\varphi} + \frac{D}{J}\,\varphi = 0 \qquad T = 2\,\pi\sqrt{\frac{D}{J}} \qquad \varphi = \frac{M_T\,l}{G\,I_{POL}}$$

Cylinder:

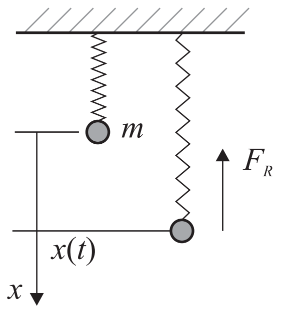
$$\tau = \frac{M_T}{0.196\,d^3} \qquad \varphi = \frac{M_T\,l}{0.089\,G\,d^4}$$

4.2 Mechanical Oscillation

F_I	N	internal forces
F_R	N	repelling force
F_G	N	weight force
m	kg	mass
a	$\mathrm{m/s^2}$	acceleration
k	N/m	spring constant
x	m	deflection, elongation
s_h	m	horizontal deflection
φ	rad, deg	excursion angle
ω_0	$\mathrm{s^{-1}}$	angular eigenfrequency
f	Hz	eigenfrequency
T	s	period duration
l	m	thread length
g	$\mathrm{m/s^2}$	gravitational acceleration
M_R	Nm	repelling torque
J_A	$\mathrm{kg\,m^2}$	moment of inertia referred to center of rotation
J_S	$\mathrm{kg\,m^2}$	moment of inertia referred to center of gravity
r	m	distance between center of rotation and center of gravity
α	$\mathrm{s^{-2}}$	angular acceleration
D	Nm/rad	torsional coefficient, torsional spring constant
t	s	time

Spring-Mass System

1-D free, straight, and undamped oscillation.



Condition: $\sum$ external forces $= \sum F_A = 0$

$$\sum F_I = m\,a$$

$$F_R = -k\,x \qquad F_R = m\,a$$

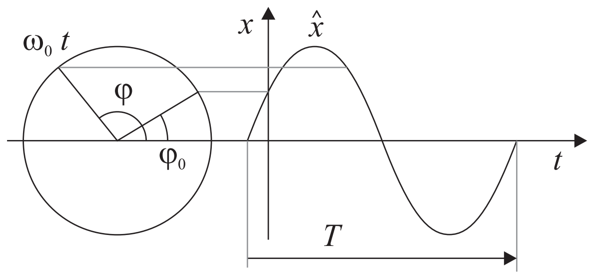
$$m\,a + k\,x = 0 \qquad m\,\ddot{x} + k\,x = 0$$

Differential equation:

$$\ddot{x} + \frac{k}{m}\,x = 0$$

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad \ddot{\omega} + \omega_0^2\,x = 0$$

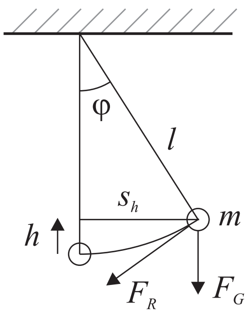
$$T = 2\,\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{2\,\pi}\sqrt{\frac{k}{m}}$$



Phase-shifted oscillation

$$x(t) = \hat{x}\,\sin\left(\omega_0\,t + \varphi_0\right)$$

Mathematical Pendulum



Horizontal deflection

$$s_h = l\,\sin\varphi \qquad F_R = F_G\,\sin\varphi$$

Differential equation:

$$\ddot{\varphi} + \frac{g}{l}\,\sin\varphi = 0$$

Condition: $\sin\varphi \approx \varphi$ for $\varphi < 5\,\mathrm{deg}$
Simplified:

$$\ddot{\varphi} + \frac{g}{l}\,\varphi = 0$$

$$\varphi(t) = \hat{\varphi}\,\cos\left(\omega_0\,t\right) \qquad \varphi(t) = \hat{\varphi}\,\cos\left(\omega_0\,t + \varphi_0\right)$$

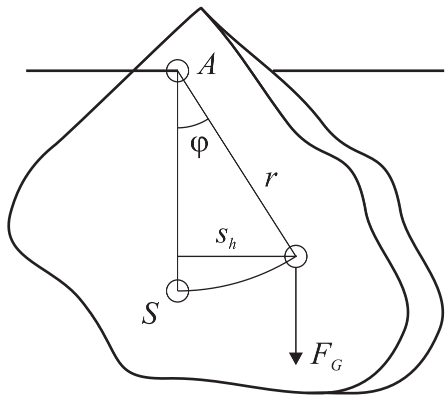
$$\omega_0 = \sqrt{\frac{k}{m}} \qquad T = 2\,\pi\,\sqrt{\frac{m}{k}}$$

$$k = \frac{F_R}{s_h} \qquad k = \frac{m\,g}{l}$$

Eigenfrequency and period duration are independent on mass:

$$\omega_0 = \sqrt{\frac{g}{l}} \qquad T = 2\,\pi\,\sqrt{\frac{l}{g}}$$

Physical Pendulum



$$M_R = \vec{r} \times \vec{F}_G \qquad M_R = r\,m\,g\,\sin\varphi$$

$$\sum M = J\,\alpha \qquad r\,m\,g\,\sin\varphi + J_A\,\alpha = 0$$

HuygensSteiner theorem:

$$J_A = J_S + m\,r^2$$

Differential equation:

$$\ddot{\varphi} + \frac{m\,g\,r}{J_A}\,\sin\varphi = 0$$

Condition: $\sin\varphi \approx \varphi$ for $\varphi < 5\,\mathrm{deg}$
Simplified:

$$\ddot{\varphi} + \frac{m\,g\,r}{J_A}\,\varphi = 0$$

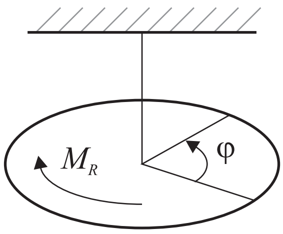
$$\varphi(t) = \hat{\varphi}\,\cos\left(\omega_0\,t\right) \qquad \varphi(t) = \hat{\varphi}\,\cos\left(\omega_0\,t + \varphi_0\right)$$

$$\omega_0 = \sqrt{\frac{m\,g\,r}{J_A}} \qquad \omega_0 = \sqrt{\frac{g}{r}}$$

$$T = 2\,\pi\,\sqrt{\frac{J_A}{m\,g\,r}} \qquad \alpha = \frac{m\,g\,r}{J_A}\,\varphi$$

$$J_S = m\,r\,\left(\frac{g\,T^2}{4\,\pi^2} - r\right)$$

Torsional Pendulum



$$M_R = \vec{r} \times \vec{F}_R \qquad M_R = J\,\alpha \qquad M_R = -D\,\varphi$$

$$\sum M = J\,\alpha \qquad \alpha = \ddot{\varphi}$$

$$J = -D\,\frac{\varphi}{\alpha} \qquad J = \frac{T^2}{4\,\pi^2}\,D$$

Differential equation:

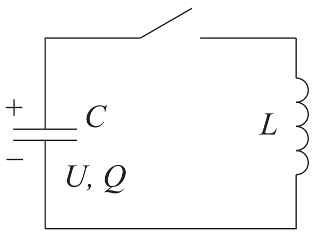
$$\ddot{\varphi} + \frac{D}{J}\,\sin\varphi = 0$$

$$\omega_0 = \sqrt{\frac{D}{J}} \qquad T = 2\,\pi\,\sqrt{\frac{J}{D}}$$

4.3 Electromagnetic Oscillation

U_C	V	voltage at the capacitor
U_L	V	voltage at the inductor
I	A	current
L	H, Vs/A	inductance
C	F, As/V	capacitance
Q	As	electric charge
q	As	momentary charge
u	V	momentary voltage
i	A	momentary current
φ	rad	phase angle
ω_0	s^{-1}	angular eigenfrequency
f	Hz	eigenfrequency
T	s	period duration
R	Ω	ohmic resistance
δ	s^{-1}	decay coefficient
ω	s^{-1}	eigenfrequency of damped oscillation
G		quality
D		damping ratio

Undamped Electromagnetic Oscillation



$$U_C = \frac{Q}{C} \qquad U_L = -L\,\frac{\mathrm{d}I}{\mathrm{d}t}$$

$$-\frac{Q}{C} - L\,\frac{\mathrm{d}I}{\mathrm{d}t} = 0$$

Differential equation:

$$\ddot{q} + \frac{q}{L\,C} = 0$$

$$q(t) = \hat{q}\,\sin\left(\omega_0\,t + \varphi_0\right) \qquad q(t) = \hat{q}\,\sin\varphi$$

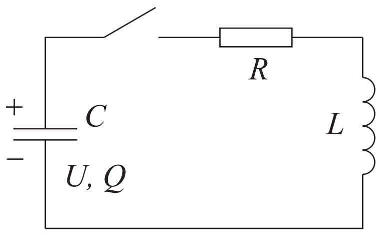
$$u(t) = \hat{u}\,\sin\left(\omega_0\,t + \varphi_0\right) \qquad u(t) = \hat{u}\,\sin\varphi$$

$$i(t) = \hat{i}\,\sin\left(\omega_0\,t + \varphi_0 - \frac{\pi}{2}\right) \qquad i(t) = \hat{i}\,\sin\left(\varphi - \frac{\pi}{2}\right)$$

$$T = 2\,\pi\,\sqrt{L\,C}$$

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \qquad L = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}Q}{\mathrm{d}t}\right) = L\,\ddot{Q}$$

Damped Electromagnetic Oscillation



Differential equation:

$$\ddot{q} + \frac{R}{L}\,\dot{q} + \frac{1}{L\,C}\,q = 0$$

$$q(t) = \hat{q}\,\mathrm{e}^{-\delta\,t}\,\sin\left(\omega_0\,t + \varphi_0\right)$$

$$\delta = \frac{R}{2\,L} \qquad \omega = \sqrt{\omega_0^2 + \delta_0^2} \qquad \omega = \sqrt{\frac{1}{L\,C} - \left(\frac{R}{2\,L}\right)^2}$$

$$G = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad G = \frac{1}{2\,D}$$

$$D = \frac{R}{2}\sqrt{\frac{C}{L}} \qquad D = \frac{\delta}{\omega_0}$$

Relationship Mechanical - Electromagnetic Oscillation

Mechan.:	deflection, elongation	x
Electr.:	charge of capacitor	q
Mechan.:	velocity	$v = \dot{x}$
Electr.:	current	$i = \dot{q}$
Mechan.:	mass	m
Electr.:	inductance	L
Mechan.:	spring constant	k
Electr.:	1 / capacitance	$1/C$
Mechan.:	damping ratio	b
Electr.:	resistance	R

Potential Energy

Mechan.: $E_{POT} = \frac{k}{2}\,x^2$ $\qquad E_{POT} = \frac{k\,x^2}{2}\,\sin^2\varphi$

Electr.: $E_{EL} = \frac{q^2}{2\,C}$

Kinetic Energy

Mechan.: $E_{KIN} = \frac{m}{2}\,v^2$ $\qquad E_{KIN} = \frac{m\,v^2}{2}\,\cos^2\varphi$

Electr.: $E_{MAG} = \frac{L\,i^2}{2}$

Undamped Oscillation

Mechan.: $x = \hat{x}\,\sin(\omega_0\,t + \varphi_0)$ $\qquad \omega_0 = \sqrt{\frac{k}{m}}$

Electr.: $q = \hat{q}\,\sin(\omega_0\,t + \varphi_0)$ $\qquad \omega_0 = \sqrt{\frac{1}{L\,C}}$

Damped Oscillation

Mechan.: $x = \hat{x}\,\mathrm{e}^{-\delta\,t}\,\sin(\omega_0\,t + \varphi_0)$ $\qquad \omega_0 = \sqrt{\omega_0^2 + \delta_0^2}$ $\qquad \delta = \frac{b}{2\,m}$

Electr.: $q = \hat{q}\,\mathrm{e}^{-\delta\,t}\,\sin(\omega_0\,t + \varphi_0)$ $\qquad \omega_0 = \sqrt{\omega_0^2 + \delta_0^2}$ $\qquad \delta = \frac{R}{2\,L}$

4.4 Types of Oscillation

F_R	N	repelling force
F_D	N	damping force, frictional force
F_E	N	exciting force
k	N/m	spring constant
m	kg	mass
a	m/s^2	acceleration
v	m/s	velocity
t	s	time
x	m	deflection, elongation
φ	rad	excursion angle
ω_0	s^{-1}	angular eigenfrequency
ω_d	s^{-1}	eigenfrequency of damped oscillation
ω_E	s^{-1}	exciting frequency, eigenfrequency in steady state
ω_{RES}	s^{-1}	resonance frequency
T_d	s	period of damped oscillation
μ		friction coefficient
b	kg/s	damping constant
δ	s^{-1}	decay coefficient
D		damping ratio
d		dissipation factor
G		quality, resonance peak
Λ		logarithmic decrement
α	rad	phase delay of resonator with respect to exciter
x_{RES}	m	resonance amplitude
x_R	m	resulting amplitude in superposition
x_{STAT}	m	static deflection at constant force
f_S	Hz	beat frequency

Free Damped Oscillation

$$F_R = m\,a \qquad F_R = -k\,x - -F_D$$

Friction independent on velocity

$$F_D = \mu\,F_N$$

Differential equation:

$$m\,\ddot{x} + \mu\,F_N + k\,x = 0$$

$$x = (\hat{x} + x_0)\,\cos(\omega_0\,t\varphi_0) - x_0$$

Friction dependent on velocity, viscous friction

$$F_D = b\,v$$

Differential equation:

$$m\,\ddot{x} + b\,\dot{x} + k\,x = 0$$

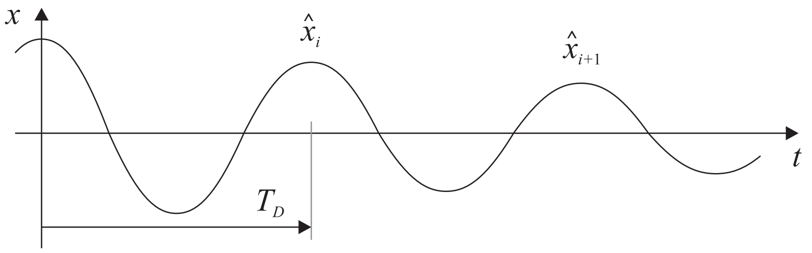
$$\ddot{x} + 2\,\delta\,\dot{x} + \omega_0^2\,x = 0 \qquad \ddot{x} + 2\,D\,\omega_0\,\dot{x} + \omega_0^2\,x = 0$$

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad \delta = \frac{b}{2\,m} \qquad D = \frac{\delta}{\omega_0} \qquad G = \frac{1}{D}$$

$$d = 2\,D \qquad d = \frac{b}{m\,\omega_0} \qquad d = \frac{b}{\sqrt{m\,k}}$$

Underdamped Case

Low damping
Conditions: $\delta < \omega_0 \quad D < 1$



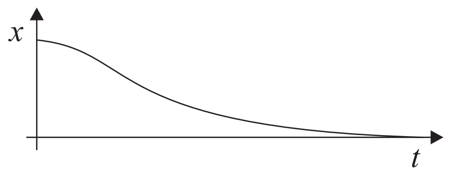
$$x(t) = \hat{x}_0\,\mathrm{e}^{-\delta\,t}\,\cos\left(\omega_d\,t + \varphi_0\right)$$

$$\omega_d = \sqrt{\frac{k}{m} - \frac{b^2}{4\,m^2}} \qquad \omega_d = \sqrt{\omega_0^2 - \delta^2} \qquad \omega_d = \omega_0^2\,\sqrt{1 - D^2}$$

$$\frac{\hat{x}_i}{\hat{x}_{i+1}} = \mathrm{e}^{-\delta\,T_d} \qquad \Lambda = \ln\frac{\hat{x}_i}{\hat{x}_{i+1}} \qquad \Lambda = \delta\,T_d$$

Aperiodic Borderline Case

Medium damping
Conditions: $\delta = \omega_0 \quad D = 1$

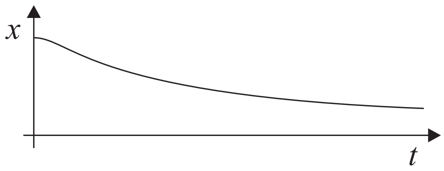


$$x(t) = (\hat{x}_0 + \hat{x}_1\,t)\,\mathrm{e}^{-\delta\,t} \qquad x(t) = \hat{x}_0\,(1 + \delta\,t)\,\mathrm{e}^{-\delta\,t}$$

$$b = 2\sqrt{m\,k}$$

Overdamped Case

Strong damping
Conditions: $\delta > \omega_0 \quad D > 1$



$$x(t) = \hat{x}_0\,\mathrm{e}^{-\delta\,t}\,\cosh(\omega_d'\,t)$$

Eigenfrequency becomes imaginary:

$$\omega_d' = j\sqrt{\delta^2 - \omega^2}$$

$$b > 2\sqrt{m\,k}$$

Forced Oscillation

Condition: $\sum$ external forces $= 0$

$$F_E + F_R + F_D = m\,\ddot{x}$$

$$F_E = \hat{F}_E\,\cos(\omega_E\,t) \qquad F_R = -k\,x$$

$$F_D = -b\,\dot{x} \qquad F_D = -b\,v$$

Differential equation:

$$m\,\ddot{x} + b\,\dot{x} + k\,x = F_E(t)$$

$$\ddot{x} = 2\,\delta\,\dot{x} + \omega_0^2\,x \qquad \ddot{x} = \frac{\hat{F}_E}{m}\,\cos(\omega_E\,t)$$

$$\alpha = \arctan\frac{\omega_E\,b}{m\,(\omega_0^2 - \omega_E^2)} \qquad \alpha = \arctan\frac{2\,\omega_E\,\delta}{\omega_0^2 - \omega_E^2}$$

$$x(t) = \hat{x}\,\omega_E\,\sin(\omega_E\,t + \varphi_0\,\omega_E) \qquad \hat{x} = \frac{\hat{F}_E}{\sqrt{(m\,\omega_0^2 - k^2) + b^2\,\omega_E^2}}$$

$$\varphi(\omega_E) = \arctan\frac{b\,\omega_E}{m\,\omega_E^2 - k}$$

Resonance

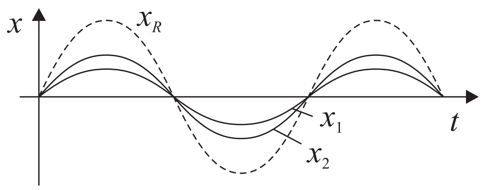
$$\omega_{RES} = \sqrt{\omega_0^2 - \frac{b^2}{2\,m^2}} \qquad \omega_{RES} = \sqrt{\omega_0^2 - 2\,\delta^2}$$

$$\hat{x}_{RES} = \frac{\hat{F}_E}{b\,\sqrt{\omega_0^2 - \delta^2}} \qquad \hat{x}_{RES} = \frac{\hat{F}_E}{b\,\omega_d^2} \qquad \hat{x}_{RES} = \frac{\hat{F}_E}{2\,\delta\,m\,\omega_d}$$

$$G = \frac{\pi\,\omega_0^2}{\Lambda\,\omega_d^2} \qquad G \approx \frac{\pi}{\Lambda}$$

$$\frac{\hat{x}}{\hat{x}_{STAT}} = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega_E^2)^2 + (2\,\delta\,\omega_E)^2}}$$

Superposition



The same orientation and the same frequency

$$\hat{x}_R = \sqrt{\hat{x}_1^2 + \hat{x}_2^2 + 2\,\hat{x}_1\,\hat{x}_2\,\cos(\varphi_{01} - \varphi_{02})}$$

$$\varphi_{0R} = \arctan\frac{\hat{x}_1\,\sin\varphi_{01} + \hat{x}_2\,\sin\varphi_{02}}{\hat{x}_1\,\cos\varphi_{01} + \hat{x}_2\,\cos\varphi_{02}}$$

Condition: $\hat{x}_1 = \hat{x}_2$

$$\hat{x}_R = 2\,\hat{x}_1\,\cos\frac{\varphi_{01} - \varphi_{02}}{2}$$

Condition: $\Delta\varphi = \pi$
Cancelling, subtraction of amplitudes

The same orientation but different frequencies: beat, low differences of frequencies

$$x_R(t) = 2\,\hat{x}\,\cos\left(\frac{\omega_1 - \omega_2}{2}\,t\right)\,\sin\left(\frac{\omega_1 + \omega_2}{2}\,t\right)$$

beat frequency

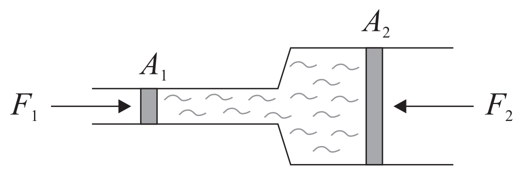
$$f_S = f_1 - f_2$$

5 Fluid Dynamics

5.1 Resting Fluid

p	Pa	pressure
p_S	Pa	gravitational pressure
p_H	Pa	air pressure at height h
p_0	Pa	air pressure at earth's surface
κ	1/Pa	compressibility
F	N	force applied to a surface
F_A	N	buoyancy force
F_G	N	weight force
F_{HUB}	N	lifting force
A	m^2	area
V	m^3	volume
ΔV	m^3	change of volume with change of pressure
W_P	J	work due to pressure
s	m	distance
h	m	height
ρ	kg/m^3	density
ρ_F	kg/m^3	density of a fluid
ρ_K	kg/m^3	density of a body inside a fluid
g	m/s^2	gravitational acceleration

Piston Pressure



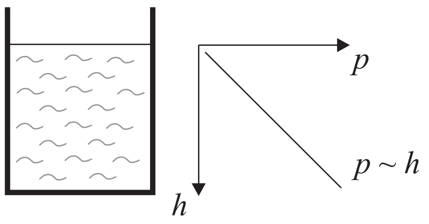
$$p = \frac{F}{A} \qquad p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$p = \frac{\mathrm{d}F}{\mathrm{d}A}$$

Work of Pressure

$$W_P = -\int \vec{F}(s)\,\mathrm{d}\vec{s} \qquad W_P = -\int_{V_1}^{V_2} \vec{p}\,\mathrm{d}V \qquad W_P = -\vec{p}\,\mathrm{d}V$$

Gravitational Pressure



$$p_S = \rho\, g\, h \qquad p_S = \frac{\rho\, g\, V}{A}$$

Gravitational pressure of gases, barometric formula:

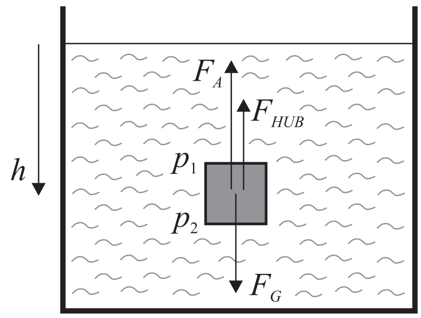
$$p_H = p_0\, \mathrm{e}^{-\frac{\rho_0\, g\, f}{p_0}}$$

Boyle's Law of Volume and Pressure

$$p_1\, V_1 = p_2\, V_2 \qquad p\, V = \text{const.}$$

$$\kappa = -\frac{1}{V}\,\frac{\mathrm{d}V}{\mathrm{d}p} \qquad \Delta V = -\kappa\, \Delta p\, V \qquad \kappa_{GAS} = \frac{1}{p}$$

Buoyancy



$$F_A = \rho_F\, V_F\, g \qquad F_A = m_F\, g = F_{G_F} \qquad F_A = \frac{\rho_F}{\rho_K}\, F_{G_K}$$

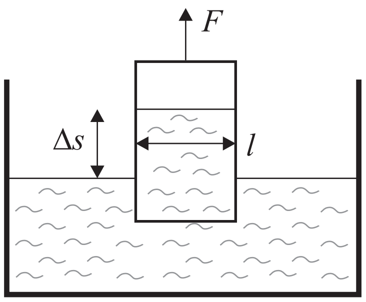
$$\Delta p = p_2 - p_1 \qquad \Delta p\, A = A\, \rho\, g\, \Delta h$$

$$F_{HUB} = F_A - F_G \qquad F_{HUB} = \rho_F\, g\, V - \rho_K\, g\, V$$

5.2 Intermolecular Forces

σ	N/m	surface tension of the fluid
W	J	work
A	m^2	area
s	m	distance
l	m	length, diameter
F	N	force perpendicular to surface
F_A	N	adhesive force (force between two substances)
F_K	N	cohesive force
p	Pa	pressure
r	m	radius of a tube
r_K	m	radius of spherical surface
α	rad, deg	contact angle
ρ	$\mathrm{kg/m^3}$	density
g	$\mathrm{m/s^2}$	gravitational acceleration

Surface Tension



$$\sigma = \frac{W}{A} \qquad \sigma = \frac{\mathrm{d}W}{\mathrm{d}A}$$

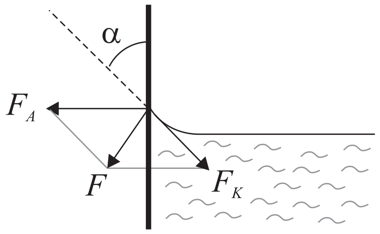
$$\sigma = \frac{F\,\Delta s}{2\,l\,\Delta s} \qquad \sigma = \frac{F}{2\,l}$$

Surface pressure of a fluid ball:

$$p = \frac{2\,\sigma}{r}$$

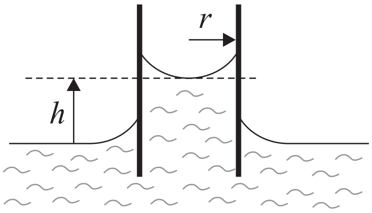
Capillarity

Wetting liquid

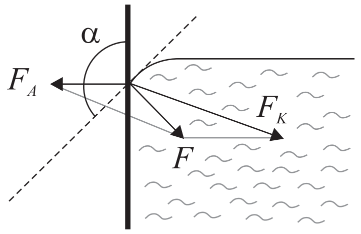


$$\alpha < 90\,\text{deg} \qquad F_A > F_K$$

Capillary ascension

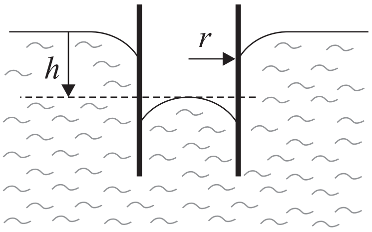


Non-wetting liquid



$$\alpha > 90\,\text{deg} \qquad F_A < F_K$$

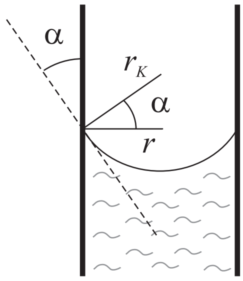
Capillary depression



$$\sigma_{GF}\,\cos\alpha = \sigma_{GW} - \sigma_{FW}$$

$$h = \frac{2\,\sigma\,\cos\alpha}{\rho\,g\,r}$$

Capillary tension



$$\Delta p = \frac{2\,\sigma}{r_K}$$

$$\cos\alpha = \frac{r}{r_K}$$

5.3 Moving Fluid

j	$kg/m^2\,s$	mass flow density
I_M	kg/s	mass flow
ρ	kg/m^3	density
v	m/s	flow velocity
v_M	m/s	average flow velocity
t	s	time
A	m^2	cross-section area
s	m	distance
m	kg	mass
V	m^3	volume
F_Z	N	pulling force
F_R	N	frictional force
F_A	N	buoyancy force
F_G	N	weight force
F_N	N	normal force, perpendicular to surface
η	Pa s	dynamic viscosity
ν	m^2/s	kinematic viscosity
h	m	distance, height
φ	$Pa^{-1}\,s^{-1}$	fluidity
Re		Reynolds number
L	m	characteristic length of a body
τ	N/m^2	shearing stress
γ	rad	gradient of shear rate
R	m	inner radius of a tube
r	m	radius of a moving cylinder
b	m	width
c	m	half plate distance
l	m	length
x	m	distance to plate
V_K	m^3	volumen of a ball
a	m/s^2	acceleration
g	m/s^2	gravitational acceleration
ω_z	s^{-1}	rotation velocity in z-direction
p	Pa	pressure
p_{ges}	Pa	total pressure
p_{STAT}	Pa	static pressure, atmospheric pressure
p_{DYN}	Pa	dynamic pressure
p_{GEOD}	Pa	geodetic pressure, gravitational pressure

Mass Flow and Density

$$j = \rho\, v \qquad j = \frac{\rho\, \mathrm{d}A\, \mathrm{d}s}{\mathrm{d}A\, \mathrm{d}t} \qquad j = \frac{\mathrm{d}Q}{\mathrm{d}A\, \mathrm{d}t}$$

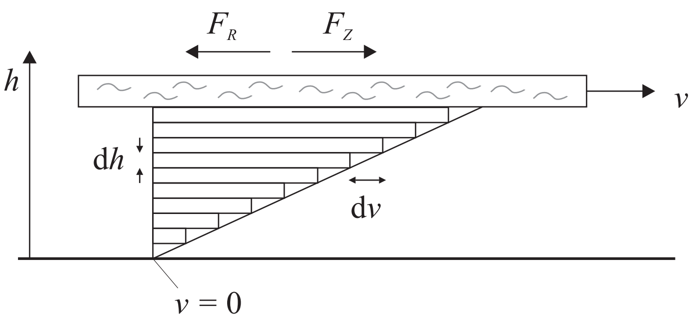
$$\mathrm{d}Q = \rho\, \mathrm{d}A\, \mathrm{d}s$$

$$I_M = \frac{\mathrm{d}m}{\mathrm{d}t} \qquad I_M = \dot{m}$$

$$I_M = \rho\, v\, A \qquad I_M = \oint_0 \rho\, v\, \mathrm{d}A \qquad I_M = \iint_A \vec{j}\, \mathrm{d}A$$

$$\dot{V} = \frac{\dot{m}}{\rho} = A\, v = \text{const.}$$

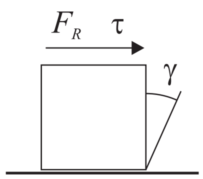
Laminar Flow, internal Friction



$$\vec{F}_Z = \eta\, A\, \frac{\mathrm{d}v}{\mathrm{d}h} \qquad \vec{F}_R = -\eta\, A\, \frac{\mathrm{d}v}{\mathrm{d}h}$$

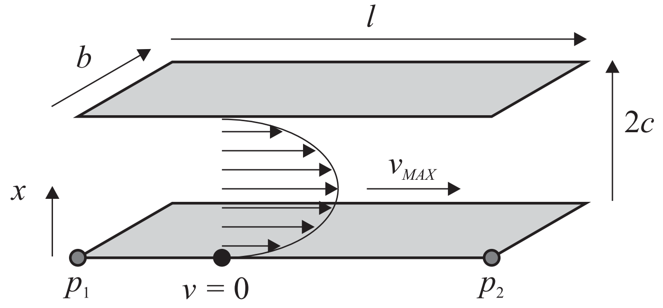
$$\eta = \frac{1}{\varphi}$$

$$Re = \frac{\rho\, L\, v}{\eta} \qquad Re = \frac{L\, v}{\nu} \qquad \nu = \frac{\eta}{\rho}$$



$$\tau = \frac{\vec{F}_R}{A} \qquad \tau = -\eta\, \dot{\gamma}$$

Laminar Flow between two Plates



$$v(x) = \frac{p_1 - p_2}{2\,\eta\, l}\,(c^2 - x^2) \qquad F_R = \frac{6\,\eta\, l\, b\, v_M}{c} \qquad v_M = \frac{1}{2\,b\,c}\,\frac{\mathrm{d}V}{\mathrm{d}t}$$

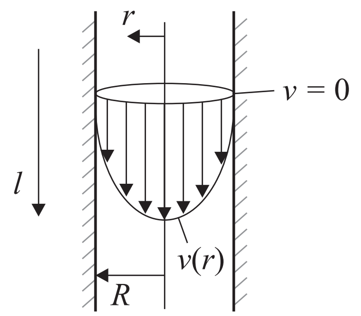
Volume flow

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{2\,b\,(p_1 - p_2)\,c^3}{3\,\eta\, l}$$

Mass flow

$$I_M = \frac{2\,b\,(p_1 - p_2)\,c^3}{3\,\eta\, l}\,\rho$$

Laminar Flow inside a Tube



$$v(r) = \frac{\pi\, R^4\,(p_1 - p_2)}{4\,\eta\, l}\,(R^2 - r^2) \qquad F_R = 8\,\pi\,\eta\, l\, v_M \qquad v_M = \frac{1}{\pi\, R^2}\,\frac{\mathrm{d}V}{\mathrm{d}t}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\pi\, R^4\,(p_1 - p_2)}{8\,\eta\, l}$$

$$I_M = \frac{\pi\, R^4\,(p_1 - p_2)}{8\,\eta\, l}\,\rho$$

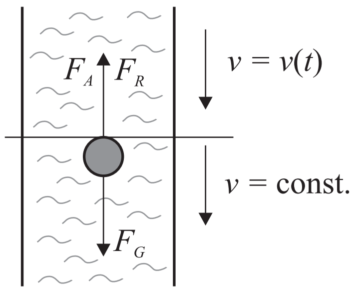
Laminar pressure loss

$$\Delta p = \frac{8\,\eta\, l}{\pi\, R^2}\,v$$

Kritical Reynolds number (above leads to turbulent viscosity inside the tube):

$$Re_{KRIT} = 2320$$

Laminar Flow surrounding a Ball



Stokes' law

$$F_R = 6\,\pi\,\eta\,Re\,v \qquad F_A = \rho_F\,V_K\,g$$

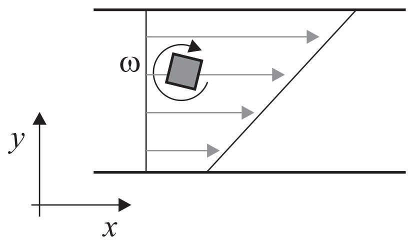
Condition: $v = v(t)$

$$m\,a = F_G - F_A - F_R$$

Condition: $v = \text{const.}$

$$0 = F_G - F_A - F_R$$

Rotation inside a Flow



$$\omega_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}$$

Condition: $v_y = 0$

$$\omega_z = -\frac{\partial v_x}{\partial y}$$

3-dimensional

$$\vec{\omega} = \text{rot}\,\vec{v} \qquad \vec{\omega} = \nabla \times \vec{v}$$

Pressure of moving Fluids

Bernoulli's equation

$$p_{ges} = p_{STAT} + p_{DYN} + p_{GEOD} = \text{const.}$$

$$p_{STAT} = \frac{F_N}{A} \qquad p_{DYN} = \frac{\rho}{2}\, v^2 \qquad p_{GEOD} = \rho\, g\, h$$

Condition: $h = \text{const.}$

$$p_{ges} = p_{STAT} + p_{DYN}$$

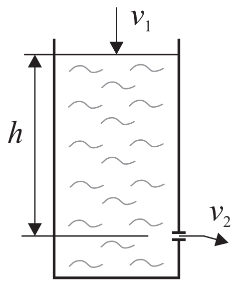
Lossy:

$$p_{ges} = p_{STAT} + p_{DYN} + p_{GEOD} + p_V$$

Continuity
Condition: $I_M = \text{const.}$

$$v_1\, A_1 = v_2\, A_2$$

Torricelli's Law of Flowing Out



Condition: $v_1 \approx 0$

$$v_2 = \sqrt{2\, g\, h} \qquad I_M = \rho\, A\, \sqrt{2\, g\, h}$$