From Walking to Running

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Abstract. The implementation of bipedal gaits in legged robots is still a challenge in state-of-the-art engineering. Human gaits could be realized by imitating human leg dynamics where a spring-like leg behavior is found as represented in the bipedal spring-mass model. In this study we explore the gap between walking and running by investigating periodic gait patterns. We found an almost continuous morphing of gait patterns between walking and running. The technical feasibility of this transition is, however, restricted by the duration of swing phase. In practice, this requires an abrupt gait transition between both gaits, while a change of speed is not necessary.

1 Introduction

A common field in robotics research is the development of autonomous systems that move like humans on two legs. Such bipedal robots impressively illustrate the progress in engineering science and bring further light into the nature of human bipedalism. A way of implementing locomotion in a biped robot is to imitate human-like leg dynamics which could result in a reduced control effort [1,2]. Dynamically, human legs show a compliant behavior in walking and running with a simple linear spring representing fundamental leg characteristics [3,4,5]. These findings support the idea of simulating legged locomotion with a highly reduced leg template, the bipedal spring-mass model with massless legs [5]. The spring-mass model can be used as a framework to investigate general control strategies for gait stabilization [6,7] or for understanding how leg design, e.g. leg segmentation [8], influences system dynamics in locomotion.

Before implementing control strategies, e.g. swing-leg control [9,7], a periodic gait pattern is required. For identifying such a gait pattern a Poincaré map [10] is applied and a single walking or running step is simulated. Here, start and end point of the simulation are equally defined and form the Poincaré section which reduces the number of independent state variables. Common events used as Poincaré section are the apex, i.e. a maximum height of the center of mass trajectory, and the touch down [10]. Both events, apex and touch down, require definitions dependent on the investigated gait. The apex in running occurs during flight phase while in walking the apex is during stance. Taken into account that more than one apex could exist within a single step [5], this event is not unique which could lead to incorrect Poincaré maps. A unique event within a single step is the touch down, however, the number of independent state variables differs between walking and running. As in walking a double support phase (both legs have ground contact at the same time) exists, the position of the previous leg is required while in running it is not (assuming that the swing leg has no influence on dynamics).

In this paper we present a Poincaré section, based on events that are unique within each step, i.e. the midstance during single support. This Poincaré section is equally defined for both walking and running. Since the term midstance is differently used in literature, we clearly define this Poincaré section as the instant where the supporting leg is oriented vertically. Using this novel map we will investigate walking and running based on purely periodic patterns. Taken into account that periodicity of gait is the basis for stabilizing swing-leg control, we believe that each of the gaits could be stabilized by a combination of swing-leg retraction and leg stiffening [7], and leg lengthening as observed in running birds [11].

The bipedal gaits have already been investigated with highly reduced models. It was discovered that walking and running are optimal at low and high speeds, respectively, due to energy expenditure [12] and self-stability [5]. However, humans can walk and run at the same speed. The purpose of this study is to explore the speed gap between walking and running as predicted by the bipedal spring-mass model. We assume that the speed gap can be smaller when not restricting to self-stability. We will investigate the technical relevance of the observed gait patterns. Here, the time required for swinging the leg forward is an important aspect limiting the realization of gaits in a robot.

2 Methods

The bipedal spring-mass model shown in Fig. 1 describes the action of the stance legs supporting a point mass m against gravity in locomotion. Both legs are represented by linear massless springs of rest length L_0 and leg stiffness k. The legs exert forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ on the point mass when in ground contact. The force of one leg directs from it's foot point $\boldsymbol{r_{fp}} = (x_{fp}, y_{fp})$ to the center of mass (COM) $\boldsymbol{r} = (x, y)$. During swing phase the leg does not influence system dynamics due to it's massless representation.

The equation of motion representing the system in the sagittal plane is

$$m\ddot{\boldsymbol{r}} = \boldsymbol{F_1} + \boldsymbol{F_2} + m\boldsymbol{g} \qquad (1)$$

where $\boldsymbol{g} = (0, -g)$ is the gravitational acceleration with $g = 9.81\text{m/s}^2$. The force of leg 1 is

$$\boldsymbol{F_1} = k \left(\frac{L_0}{|\boldsymbol{r} - \boldsymbol{r_{fp1}}|} - 1 \right) (\boldsymbol{r} - \boldsymbol{r_{fp1}}) \qquad (2)$$

as long as this leg has ground contact. If the leg length reaches it's resting length L_0 the transition from stance to swing phase occurs and the leg force is set to

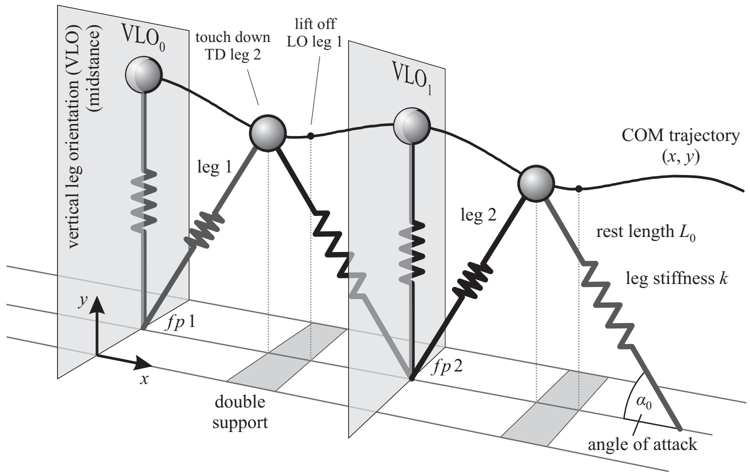

Fig. 1. Walking of the bipedal spring-mass model starting at the instant of VLO (vertical leg orientation).

zero. The transition from swing to stance happens when the landing condition $y_{TD} = L_0 \sin(\alpha_0)$ is fulfilled, where α_0 is the angle of attack.

The system parameters are based on human dimensions, i.e. body mass $m =$ 80kg and leg length $L_0 = 1$m. The model can be compared with other bipeds using dimensional analysis. Due to the models simplicity, Geyer et al. [5] showed that it has only three fundamental parameters, i.e. the dimensionless leg stiffness $\widetilde{k} = kL_0/(mg)$, the angle of attack α_0, and the dimensionless system energy $\widetilde{E} = E/(mgL_0)$. The energy E is the constant total energy of the conservative model.

The model shows walking and running in a steady-state manner which means that strides are periodically repeated. Such a gait pattern is completely described by the selected system parameters (k, α_0, E) and initial conditions (x_0, y_0, v_{x0}, v_{y0}). In this study the initial conditions are chosen such that one leg has ground contact and is vertically orientated. Clearly, the COM is orientated vertically above center of pressure $x = x_{fp1}$. A single step of a gait is completed when the counter leg has single support and is vertically orientated where $x = x_{fp2}$ is valid. The instant of Vertical Leg Orientation (VLO) exists in both gaits, walking and running, hence, it is independent on gait. With these definitions we investigate locomotion by analyses of VLO return maps, i.e. Poincaré maps with VLO as Poincaré section (visualized in Fig. 1).

Using VLO as starting point we can reduce the number of independent initial conditions to y_0 and v_{y0} due to the following reasons. The horizontal position x_0 is always zero with respect to the center of pressure x_{fp} and only one leg has ground contact. Since the system is conservative, the horizontal velocity v_{x0} at VLO depends on height y_0 and vertical velocity v_{y0}:

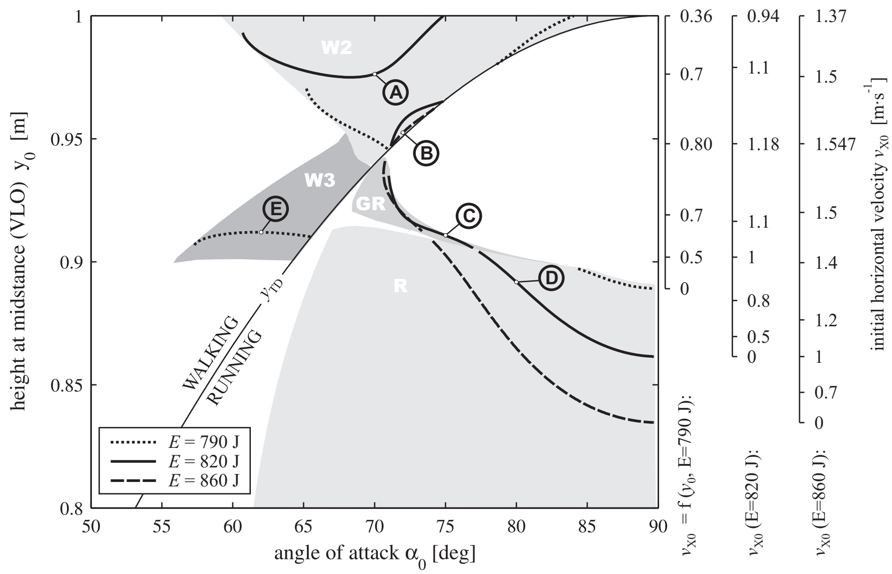

Fig. 2. Initial conditions (systems state at VLO y_0, $v_{x0} = \text{f}(y_0, E)$) of symmetric gait patterns ($v_{x0} = 0$) for three selected system energies E and one leg stiffness $k = 15\,\text{kN/m}$. Each point on the thick lines represents a periodic gait solution. Five representative gait patterns (A-E) are shown in detail in Fig. 3. The gray areas indicate the regions where running (R), grounded running (GR), and walking (W2 and W3) were found for $750\text{J} < E < 2000\text{J}$. W2 and W3 represent regions where the ground reaction force has normally two and three humps, respectively. The thin black line represents the landing height $y_{TD} = L_0 \sin(\alpha_0)$ where touch down (TD) takes place.

$$v_{x0} = \sqrt{\frac{2}{m}\left(E - m\,g\,y_0 - \frac{k}{2}(L_0 - y_0)^2 - \frac{m}{2}v_{y0}^2\right)} \tag{3}$$

In this paper we investigate symmetric gaits only. Here, the COM trajectory can be mirrored about midstance and VLO is identical to midstance while v_{y0} is zero. A Newton-Raphson algorithm is used to find periodic gait solutions.

3 Results

With the VLO return map we found several solutions of periodic gaits (Fig. 2). We distinguish between walking and running gaits by observing the vertical excursion during stance phase. In case of walking the COM is lifted during single stance phase, as can be suggested by a VLO height y_0 above touch down level y_{TD}. In running gaits the COM is lowered during stance as illustrated with pattern D in Fig. 3. Here, the VLO height is below y_{TD} (Fig. 2).

We categorized walking patterns by counting maxima in the ground reaction force (GRF) of representative solutions. However, in some cases the gait patterns

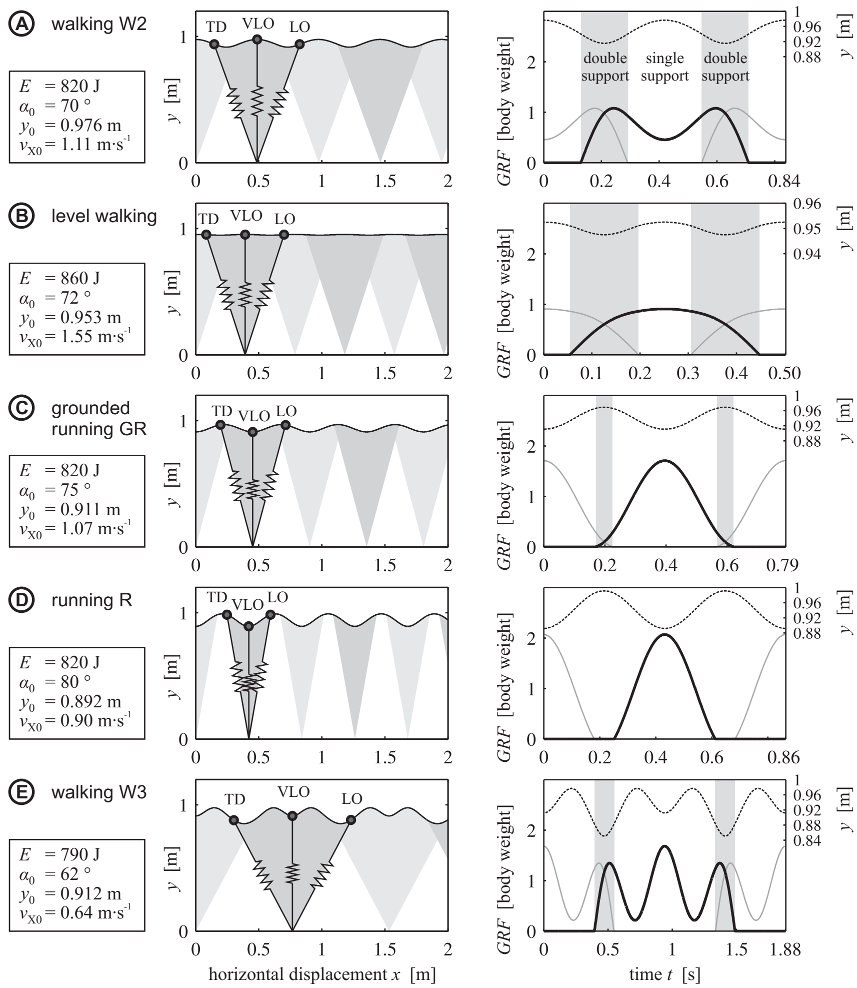

Fig. 3. Representative gait patterns of the bipedal spring-mass model related to the selected initial conditions in Fig. 2. The left column shows leg positions at touch down (TD), VLO, and lift off (LO). The graphs on the right side show vertical ground reaction forces (GRF) and the vertical excursion of the COM (dotted lines) during a complete gait cycles (two steps).

slightly transform into patterns with less maxima. Here, the larger number of maxima was taken into account for the categorization. An example is pattern B which has a single peak in the GRF while it's directly connected with patterns having two maxima.

For running we found only one significant maximum of the GRF. Normally, running is defined as a gait including flight phases (pattern D). However, there exists a neighboring gait with the same vertical excursion of the COM trajectory having a double support phase like in walking (pattern C). Due to the mentioned definition of walking and running we classify it as a running gait, and since the system has always ground contact we call it grounded running (GR).

In Fig. 2 is shown that neither walking gaits nor running gaits cross the touch down height $y_{TD} = L_0 \sin(\alpha_0)$. Examining the solutions more carefully, we find that walking (W2) seems to be connected with running (R) via grounded running (GR). Observing the lines of $E = 860$J starting at pattern B and following to GR we find just a small vertical gap at $\alpha_0 \approx 71$deg. Between GR and R a small gap at $\alpha_0 \approx 73.5$deg is found as well.

The walking and running gait overlap, regarding average velocity, is shown in Fig. 4. At low speeds ($\approx$ 1m/s) walking requires a flatter angle of attack than running and grounded running. With increasing speed both, walking and running, overlap with the angle of attack as well. Additionally to the speed the swing time (i.e. the time from lift off (LO) to touch down (TD)) is visualized. In walking (Fig. 4(a)) the swing time decreases with increasing angle of attack and speed. Very slow walking, having more than two force peaks (W3 and W4), contains swing durations that are longer than 0.7s. In fast walking (e.g. gait pattern B) the time between lift off and touch down is less than 0.2s. In contrast to walking, in running the swing time increases with increasing angle of attack and speed. Here, the system has more time to swing the leg forward, i.e. more than 0.25s.

4 Discussion

One purpose of this study was to explore the speed gap between walking and running. In contrast to previous studies [5,12] both gaits are possible at low and medium speeds (Fig. 4), hence, a speed gap does not exist. Moreover, walking is almost directly connected with running via grounded running alias jogging. This connection leads to the assumption that a smooth transition from walking to running is possible. However, the swing time in the neighborhood of the mentioned connection ($\diamond$ in Fig. 4) is very low. With less than 0.3s it might be technically not applicable and the natural dynamics of a pendulum-like swing leg cannot be used. Hence, the robot should jump over this parameter region when changing from walking to running. This model prediction supports the need of an abrupt gait transition within one or two steps as observed in humans. As walking shows a smaller vertical amplitude of the COM trajectory compared to running (Fig. 3), the COM has to be lowered and lifted significantly within the transition step(s) to match the aimed running pattern. This might be achieved by a softer leg spring at the transition step(s).

The model predicts that walking can be faster than the preferred gait transition speed ($\approx$ 2m/s). This gait shows almost no vertical motion (pattern B in Fig. 3) which relates to 'level walking' [13]. Due to the very short swing duration with less than 0.1s above 2m/s it is neither technically nor biologically feasible (in humans a swing time of $\approx$ 0.3s is observed [14]). The limitation of walking speed might be caused by the linear force-length relationship of the leg spring.

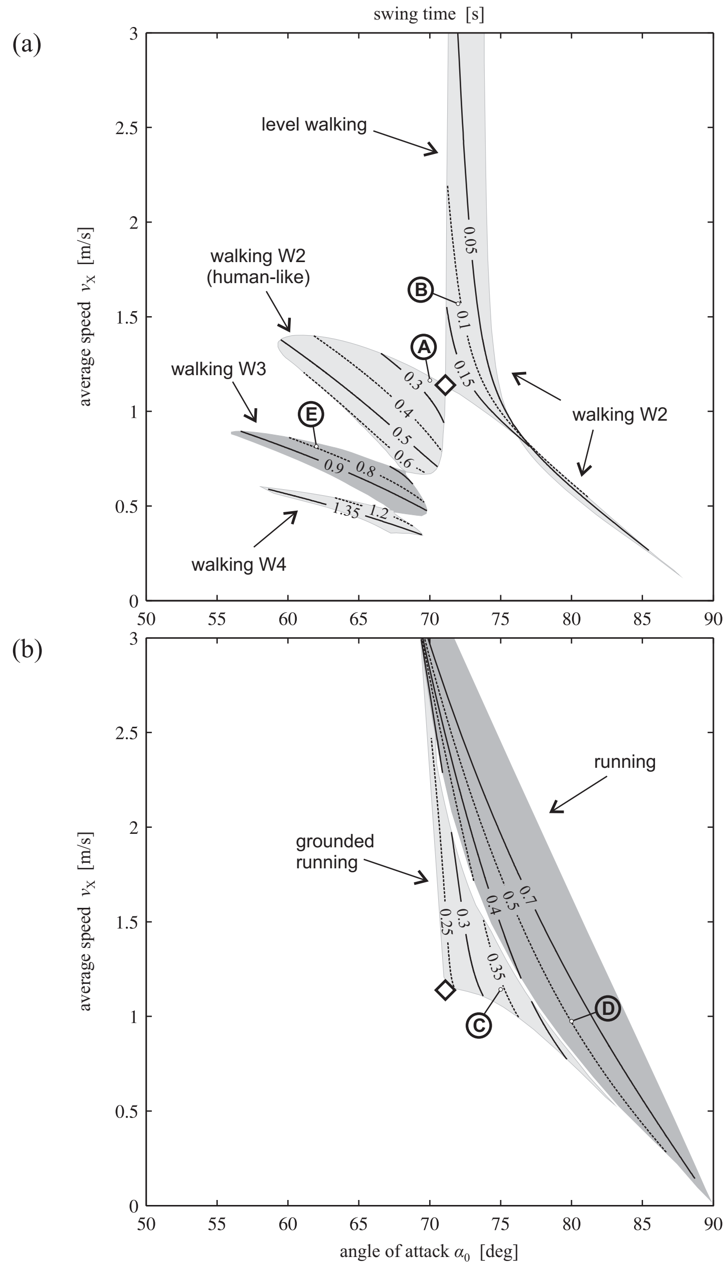

Fig. 4. Average speed and swing time (isolines) of selected gaits. Due to coexistence in similar regions, walking and running gaits were separated into two diagrams, (a) and (b), respectively. The symbol $\diamond$ is a landmark representing a bifurcation inside W2 and connects the region of W2 with that of grounded running. The patterns A-E correspond to gait solutions in Fig. 2 and 3.

We assume that another leg characteristic is needed to overcome this drawback. Here, a leg with increasing or decreasing stiffness function [8] might help.

In this study we presented the VLO return map as a method for analyzing locomotion. This map is neither restricted to gait nor to symmetry. Furthermore, we have successfully tested the VLO as Poincaré section in higher dimensional models, i.e. in a system with distributed body mass and in a quadruped. In order to find periodic gait patterns in these models we inherited solutions from the lower dimensional spring-mass model, presented here. Due to the presentation in Cartesian coordinates, walking and running can be clearly distinguished. In walking the body is lifted during stance phase, indicated by a VLO height above the touch down level. The opposite behavior is found in running.

In further studies the walking patterns will be implemented into our bipedal robot testbed (PogoWalker) to prove the model predictions.

Acknowledgments

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